**FPT University**

**Course: Discrete mathematics**

**Course ID: MAD101**

**Student’s name: Group:**

**EXERCISES – CHAPTER 3**

**PART I (7 MARKS)**

1. Devise an algorithm that ﬁnds all terms of a ﬁnite sequence of integers that are greater than the sum of all previous terms of the sequence.
2. Find the least integer n such that f(x) is **O(xn)** for each of these functions.
3. f(x) = 2x2 + x3 log x
4. f(x) = 3x5 + (log x8)
5. f(x) =
6. f(x) = (x3 + 5 log x)/(x2 + 1)
7. Give a **big-O estimate** for each of these functions. For the function g in your estimate f(n) is O(g(n)), use a simple function g of smallest order.
8. (n3+n2 logn)(log n+1) + (17 logn+19)(n3+2)
9. n log(n2 + 1) + n2 logn
10. (n logn + 1)2 + (logn + 1)(n2 + 1)
11. (2n + n2)(n3 + 3n)
12. For each of these functions, determine whether that function is **(x2)** and whether it is **(x2).**
13. f(x) = 2x2 + xlogx
14. f(x) = x + x2logx
15. f(x) = 4x + 17
16. Find the **best big-O** for the algorithm

Procedure H(n: positive integer)

For i: = 1 to n do

For j: = 1 to n do

Print “Hello”

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1. What are the **quotient** and **remainder** when
2. 777 is divided by 21?
3. −1 is divided by 23?
4. −2002 is divided by 87?
5. 243 is divided by 420?
6. 100! is divided by 99!?
7. Let m be a positive integer. Show that a + b ≡ 2b (mod m) if a ≡ b(mod m).
8. Evaluate these quantities.
9. −17 mod 2
10. -144 div 7
11. −101 mod 13
12. 199 div 19
13. Decide whether each of these integers is **congruent** to 3 modulo 7.
14. 37
15. 66
16. −17
17. −67
18. Find the **base b expansion** of the number 97 if
19. b = 3
20. b = 5
21. b = 7
22. Convert the **base 13 expansion** of each of these integers to a **decimal expansion**.
23. (17)13
24. (A3)13
25. (1A2B)13
26. Find the **prime factorization** of each of these integers.
27. 88
28. 126
29. 729
30. 1001
31. 1111
32. Which positive integers less than 12 are **relatively prime** to 12?
33. Use the **Euclidean algorithm** to ﬁnd
34. gcd(12, 18)
35. gcd(148, 48)
36. A sequence of **pseudorandom numbers** is generated using the pure multiplicative generator *xn*  1  (3*xn* **+ 5) mod** 11 with some seed x0. Suppose x3 = 6 find x4 and x2.
37. Suppose that a computer has only the memory locations 01 2*…*29. Use the **hashing function** *h* where *h*(*x*)  (*x*  3) **mod** 30 to determine the memory locations in which 77 and 97 are stored.
38. A message has been **encrypted** using the function *f* (*x*)  (*x*  7) **mod** 26. If the message in coded form is NVVK PVI, **decode** the message.
39. **Encode** the message “I LOVE YOU” using the function *f* (*x*)  (*x*  6) **mod** 26.

**PART II (3 MARKS)**

1. A **perfect number** is a positive integer that is equal to the sum of its proper positive divisors. For example, 6 (= 1 + 2 + 3) is a perfect number. Devise an algorithm that ﬁnds all perfect numbers less than 1000.
2. Find the “best” big-oh notation to describe the complexity of the algorithm.
3. A **binary search** of *n* elements.
4. A **linear search** to find the smallest number in a list of *n* numbers.
5. An algorithm that lists all ways to put the numbers 123*…**n* in a row.
6. An algorithm that prints all bit strings of length *n.*
7. An iterative algorithm to compute *n*, (counting the number of multiplications).
8. Show that if a | b and b | a, where a and b are integers, then a = b or a = −b.
9. Show that if a, b, and c are integers, where a ≠ 0 and c ≠ 0, such that ac | bc, then a | b.
10. Find the sum of each of these pairs of numbers in base 3 expansion.
11. (112)3 + (210)3
12. (2112)3 +(12021)3
13. (20001)3 +(1111)3
14. The value of the Euler φ-function at the positive integer n is deﬁned to be the number of positive integers less than or equal to n that are relatively prime to n. Find these values of the Euler φ-function.
15. φ(4).
16. φ(13).
17. φ(n) where n is prime
18. **Parity Check Bits.** Digital information is represented by bit string, split into blocks of a speciﬁed size. Before each block is stored or transmitted, an extra bit, called a parity check bit, can be appended to each block. The parity check bit xn+1 for the bit string x1x2 ...xn is deﬁned by xn+1 = x1 + x2 +···+ xn mod 2. It follows that xn+1 is 0 if there are an even number of 1 bits in the block of n bits and it is 1 if there are an odd number of 1 bits in the block of n bits. When we examine a string that includes a parity check bit, we know that there is an error in it if the parity check bit is wrong. However, when the parity check bit is correct, there still may be an error. A parity check can detect an odd number of errors in the previous bits, but not an even number of errors.

Suppose you received these bit strings over a communications link, where the last bit is a parity check bit. In which string are you sure there is an error?

a) 00000111111

b) 10101010101

c) 11111100000

1. **UPCs**. Retail products are identiﬁed by their Universal Product Codes (UPCs). The most

common form of a UPC has 12 decimal digits: the ﬁrst digit identiﬁes the product category, the next ﬁve digits identify the manufacturer, the following ﬁve identify the particular product, and the last digit is a check digit. The check digit is determined by the congruence

3x1 + x2 + 3x3 + x4 + 3x5 + x6 + 3x7 + x8 + 3x9 + x10 + 3x11 + x12 ≡ 0 (mod 10).

Determine the check digit for the UPCs that have these initial 11 digits.

a) 73232184434

b) 63623991346

1. **ISBNs.** All books are identiﬁed by an International Standard Book Number (ISBN-10), a 10-digit code x1x2 ...x10, assigned by the publisher. An ISBN-10 consists of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and ﬁnally, a check digit that is either a digit or the letter X (used to represent 10). This check digit is selected so that

The ISBN-10 of the sixth edition of Elementary Number Theory and Its Applications is 0-321-500Q1-8, where Q is a digit. Find the value of Q.

**THE END**